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Derivation of Two-Dimensional (2-D) Conduction Equation in Generalized Coordinates With Constant and Anisotropic Physical Properties

Paul J. Conroy

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Paul J. Conroy

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U.S. Army Research Laboratory
ATTN: AMSRL-WT-PA
Aberdeen Proving Ground, MD 21005-5066

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13. ABSTRACT *(Maximum 200 words)*

The purpose of this paper is to provide a working report for potential future modifications to existing heat conduction codes. A generalized form of the two-dimensional (2-D) arbitrary geometry axisymmetric heat conduction equation has been derived from first principles. This has been further generalized by allowing nonidealized anisotropic behavior of the material properties.

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1. INTRODUCTION

This report concerns the derivation of the two-dimensional (2-D) heat conduction equation in generalized axisymmetric coordinates for both constant and nonidealized anisotropic material properties. The purpose of this report is to provide a working paper for potential future modifications to existing heat conduction codes.

2. DERIVATION OF 2-D HEAT CONDUCTION EQUATION

Beginning with a control volume description in normal coordinates as shown in Figure 1 and applying the typical Taylor series expansion to Fourier's heat conduction law over the control volume enables one to preform the energy balance. The cross-sectional areas and volume of the control volume in the axisymmetric coordinate system are

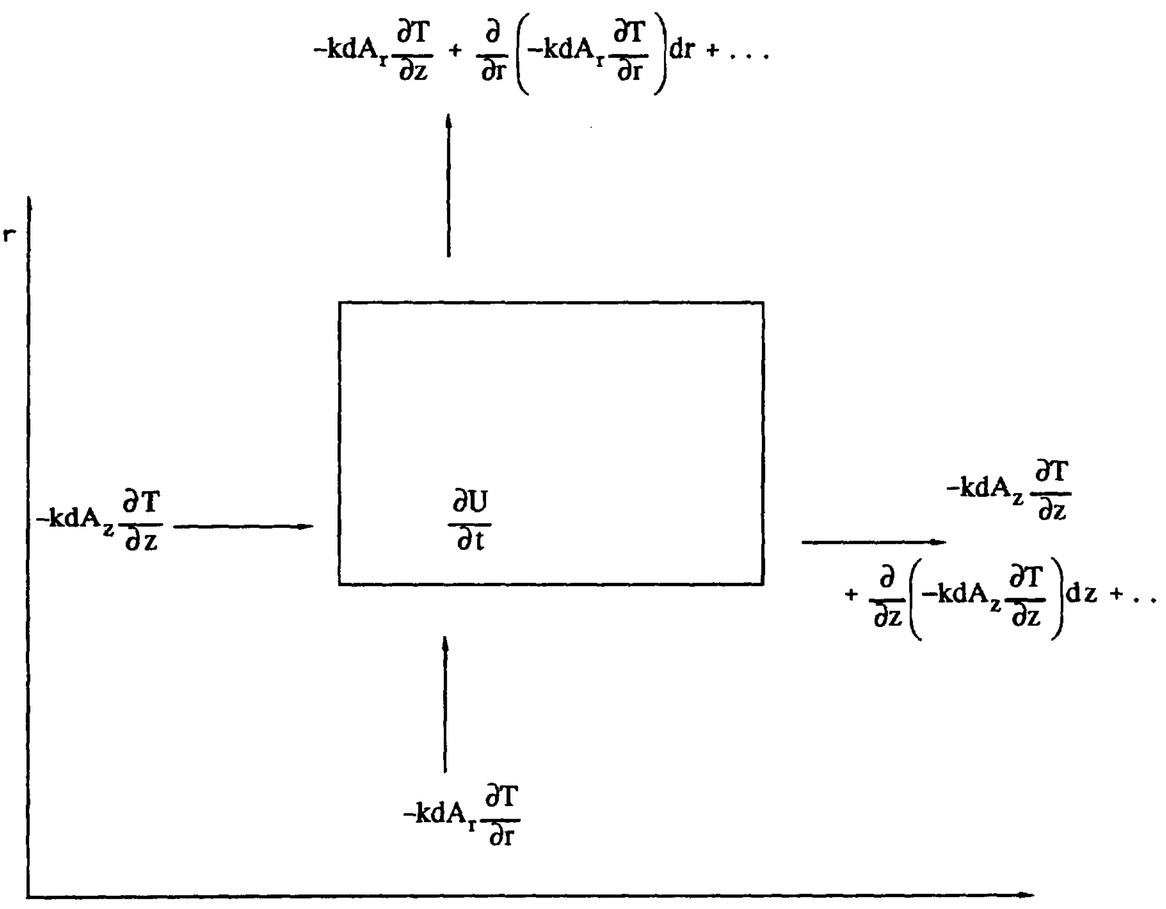

Figure 1. Energy balance control volume.

$$dA_r = rd\theta dz \ ,$$

$$dA_z = rd\theta dr \ ,$$

and

$$dV = rd\theta drdz \ . \tag{1}$$

Balancing the energy produces the cylindrical heat conduction equation through the following steps. Balancing the stored energy with the difference in the flux across the boundaries produces

$$\begin{aligned} dV\frac{\partial u}{\partial t} &= kdA_r \frac{\partial T}{\partial r} + kdA_z \frac{\partial T}{\partial z} - kdA_z \frac{\partial T}{\partial z} - kdA_r \frac{\partial T}{\partial r} \\ &- \frac{\partial}{\partial r}\left(-kdA_r \frac{\partial T}{\partial r}\right)dr - \frac{\partial}{\partial}\left(-kdA_z \frac{\partial T}{\partial z}\right)dz \ , \\ &= \frac{\partial}{\partial r}\left(krd\theta dz \frac{\partial T}{\partial r}\right)dr + \frac{\partial}{\partial z}\left(krd\theta dr \frac{\partial T}{\partial z}\right)dz \ , \\ &= \frac{\partial}{\partial r}\,(krd\theta dz)\left(\frac{\partial T}{\partial r}\right)dr + krd\theta dzdr \frac{\partial^2 T}{\partial r^2} \\ &+ \frac{\partial}{\partial z}\,(krd\theta dr)\left(\frac{\partial T}{\partial z}\right)dz + krd\theta drdz \frac{\partial^2 T}{\partial z^2} \ , \end{aligned} \tag{2}$$

where

$$\frac{\partial U}{\partial t} = \frac{\partial U}{\partial T}\frac{\partial T}{\partial t} = \rho C_p \frac{\partial T}{\partial t} \ . \tag{3}$$

Assuming constant properties reduces this to

$$\rho C_p d\theta dzdr \frac{\partial T}{\partial t} = kd\theta dzdr \left(\frac{\partial T}{\partial r}\right)dr + krd\theta dzdr \frac{\partial^2 T}{\partial r^2} + krd\theta drdz \frac{\partial^2 T}{\partial z^2} \ . \tag{4}$$

Dividing through by kdV produces the familiar constant property cylindrical heat conduction equation

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2}\,, \tag{5}$$

where the diffusivity, α, is

$$\alpha = \frac{k}{\rho C_p}\,. \tag{6}$$

3. TRANSFORMATION TO GENERALIZED COORDINATES

Transformation of the orthogonal cylindrical coordinate system to a more generalized coordinate system with body geometry included is preformed to allow for a body-contoured grid scheme while maintaining orthogonality for computational purposes. Demonstrated in Figure 2 is a generalized axisymmetric body with global coordinate system of r and z as well as the local body coordinate system of η and ξ. The inner and outer surfaces are defined by $R_i(z)$ and $R_o(z)$ respectively.

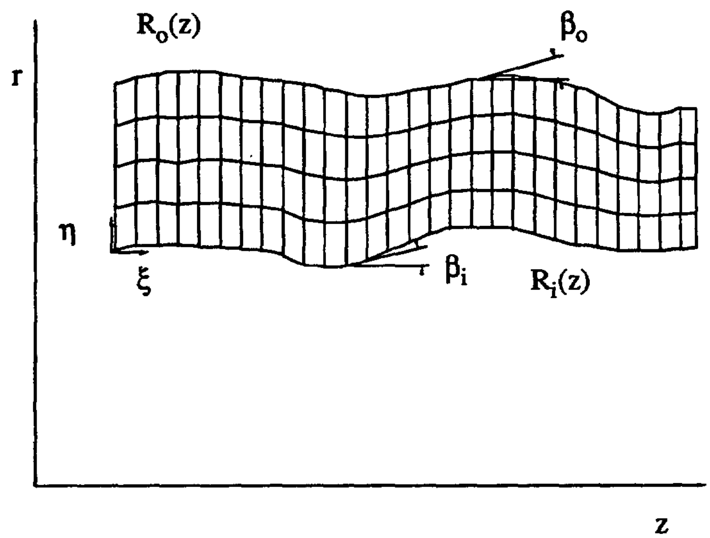

Figure 2. General axisymmetric body.

The transformations from r, z, θ, to η, ξ, ϕ are

$$\eta = \frac{r - R_i(z)}{R_o(z) - R_i(z)} , \tag{7}$$

$$\xi = z \tag{8}$$

and

$$\phi = \theta . \tag{9}$$

The local thickness of the body is represented as

$$tk = R_o(z) - R_i(z) . \tag{10}$$

Writing out the chain rule for each of the diffusion terms of equation (5) in the transformed region with Ψ being a scaler potential function (for instance, temperature is a scaler potential function) produces the following terms:

$$\frac{\partial \Psi}{\partial r} = \frac{\partial \Psi}{\partial \eta}\frac{\partial \eta}{\partial r} + \frac{\partial \Psi}{\partial \xi}\frac{\partial \xi}{\partial r} , \tag{11}$$

$$\frac{\partial \Psi}{\partial z} = \frac{\partial \Psi}{\partial \eta}\frac{\partial \eta}{\partial z} + \frac{\partial \Psi}{\partial \xi}\frac{\partial \xi}{\partial z} , \tag{12}$$

$$\begin{aligned}
\frac{\partial^2 \Psi}{\partial r^2} &= \frac{\partial}{\partial r}\left(\frac{\partial \Psi}{\partial \eta}\frac{\partial \eta}{\partial r}\right) + \frac{\partial}{\partial r}\left(\frac{\partial \Psi}{\partial \xi}\frac{\partial \xi}{\partial r}\right) \\
&= \frac{\partial^2 \eta}{\partial r^2}\frac{\partial \Psi}{\partial \eta} + \frac{\partial \eta}{\partial r}\frac{\partial}{\partial r}\left(\frac{\partial \Psi}{\partial \eta}\right) + \frac{\partial \Psi}{\partial \xi}\frac{\partial^2 \xi}{\partial r^2} + \frac{\partial \xi}{\partial r}\frac{\partial}{\partial r}\left(\frac{\partial \Psi}{\partial \xi}\right) \\
&= \frac{\partial^2 \eta}{\partial r^2}\frac{\partial \Psi}{\partial \eta} + \left(\frac{\partial \eta}{\partial r}\right)^2 \frac{\partial^2 \Psi}{\partial \eta^2} + \frac{\partial \eta}{\partial r}\frac{\partial \xi}{\partial r}\frac{\partial}{\partial \xi}\left(\frac{\partial \Psi}{\partial \eta}\right) + \frac{\partial \Psi}{\partial \xi}\frac{\partial^2 \xi}{\partial r^2} \\
&+ \left(\frac{\partial \xi}{\partial r}\right)^2 \frac{\partial^2 \Psi}{\partial \xi^2} + \frac{\partial \eta}{\partial r}\frac{\partial \xi}{\partial r}\frac{\partial}{\partial \eta}\left(\frac{\partial \Psi}{\partial \xi}\right) ,
\end{aligned} \tag{13}$$

and

$$\frac{\partial^2\Psi}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial\Psi}{\partial\eta}\frac{\partial\eta}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial\Psi}{\partial\xi}\frac{\partial\xi}{\partial z}\right)$$

$$= \frac{\partial^2\eta}{\partial z^2}\frac{\partial\Psi}{\partial\eta} + \frac{\partial\eta}{\partial z}\frac{\partial}{\partial z}\left(\frac{\partial\Psi}{\partial\eta}\right) + \frac{\partial\Psi}{\partial\xi}\frac{\partial^2\xi}{\partial z^2} + \frac{\partial\xi}{\partial z}\frac{\partial}{\partial z}\left(\frac{\partial\Psi}{\partial\xi}\right)$$

$$= \frac{\partial^2\eta}{\partial z^2}\frac{\partial\Psi}{\partial\eta} + \left(\frac{\partial\eta}{\partial z}\right)^2\frac{\partial^2\Psi}{\partial\eta^2} + \frac{\partial\eta}{\partial z}\frac{\partial\xi}{\partial z}\frac{\partial}{\partial\xi}\left(\frac{\partial\Psi}{\partial\eta}\right) + \frac{\partial\Psi}{\partial\xi}\frac{\partial^2\xi}{\partial z^2}$$

$$+ \left(\frac{\partial\xi}{\partial z}\right)^2\frac{\partial^2\Psi}{\partial\xi^2} + \frac{\partial\eta}{\partial z}\frac{\partial\xi}{\partial z}\frac{\partial}{\partial\eta}\left(\frac{\partial\Psi}{\partial\xi}\right). \tag{14}$$

The following simplifications are used to reduce the transformed terms

$$\frac{\partial\eta}{\partial r} = \frac{1}{tk}, \tag{15}$$

$$\frac{\partial\xi}{\partial r} = 0, \tag{16}$$

$$\frac{\partial^2\eta}{\partial r^2} = 0$$

$$\frac{\partial\eta}{\partial z} = 1, \tag{17}$$

$$\frac{\partial\eta}{\partial z} = -\frac{1}{tk}\left(\frac{dR_i}{dz} + \eta\frac{d(tk)}{dz}\right), \tag{18}$$

and

$$\frac{\partial^2\eta}{\partial z^2} = \frac{L}{tk^2}\,\frac{d(tk)}{dz}$$

$$- \frac{1}{tk}\left(\frac{d^2R_i}{dz^2} + \eta\,\frac{d^2(tk)}{dz^2} - \frac{L}{tk}\,\frac{d(tk)}{dz}\right). \qquad (19)$$

For consolidation, let

$$L = \left(\frac{dR_i}{dz} + \eta\,\frac{d(tk)}{dz}\right). \qquad (20)$$

Substituting the previous terms into equations (11) through (14) results in the following transformed diffusion terms:

$$\frac{\partial\Psi}{\partial r} = \frac{1}{tk}\,\frac{\partial\Psi}{\partial\eta}\,, \qquad (21)$$

$$\frac{\partial^2\Psi}{\partial r^2} = \frac{1}{tk^2}\,\frac{\partial^2\Psi}{\partial\eta^2}\,, \qquad (22)$$

$$\frac{\partial\Psi}{\partial z} = \frac{\partial\Psi}{\partial\xi} - \frac{L}{tk}\,\frac{\partial\Psi}{\partial\eta}\,,$$

$$\frac{\partial^2\Psi}{\partial z^2} = \frac{\partial^2\Psi}{\partial\xi^2} + \frac{d}{dz}\left(\frac{-L}{tk}\right)\left(\frac{\partial\Psi}{\partial\eta}\right)$$

$$+ \frac{L^2}{tk^2}\,\frac{\partial^2\Psi}{\partial\eta^2} - \frac{L}{tk}\left[\frac{\partial}{\partial\eta}\left(\frac{\partial\Psi}{\partial\xi}\right) + \frac{\partial}{\partial\xi}\left(\frac{\partial\Psi}{\partial\eta}\right)\right], \qquad (23)$$

and

$$= \frac{\partial^2\Psi}{\partial\xi^2} + \frac{L}{tk^2}\,\frac{d(tk)}{dz}$$

$$+ \frac{-1}{tk}\left(\frac{d^2R_i}{dz^2} + \eta\frac{d^2(tk)}{dz^2} - \frac{L}{tk}\,\frac{d(tk)}{dz}\right)\frac{\partial\Psi}{\partial\eta}$$

$$+ \frac{L^2}{tk^2}\,\frac{\partial^2\Psi}{\partial\eta^2} - \frac{L}{tk}\left[\frac{\partial}{\partial\eta}\left(\frac{\partial\Psi}{\partial\xi}\right) + \frac{\partial}{\partial\xi}\left(\frac{\partial\Psi}{\partial\eta}\right)\right]. \tag{24}$$

Substituting diffusion terms, equations (21) through (24), into the original conduction equation results in the following transformed conduction equation with temperature as the potential function. This relation is valid for any arbitrary axisymmetric shape with constant material properties, where $0 \le \eta \le 1$.

$$\frac{1}{\alpha}\,\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial\xi^2} + \frac{L}{tk^2}\,\frac{d(tk)}{dz}\left(\frac{\partial T}{\partial\eta}\right)$$

$$- \frac{1}{tk}\left(\frac{d^2R_i}{dz^2} + \eta\frac{d^2(tk)}{dz^2} - \frac{L}{tk}\,\frac{d(tk)}{dz}\right)\frac{\partial T}{\partial\eta}$$

$$+ \frac{L^2}{tk^2}\,\frac{\partial^2 T}{\partial\eta^2} - \frac{L}{tk}\left[\frac{\partial}{\partial\eta}\left(\frac{\partial T}{\partial\xi}\right) + \frac{\partial}{\partial\xi}\left(\frac{\partial T}{\partial\eta}\right)\right]$$

$$+ \frac{1}{tk^2}\,\frac{\partial^2 T}{\partial\eta^2} + \frac{1}{tk\big((\eta)tk + R_i\big)}\,\frac{\partial T}{\partial\eta}. \tag{25}$$

Regrouping the right-hand side in terms of the temperature derivatives, replacing the variable, transforming ζ to Z, L, and noting that the cross derivatives are the same for a well-behaved function leaves the following (relatively) concise form

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2}$$

$$- \frac{1}{tk}\left(\frac{d^2R_i}{dz^2} + \eta\frac{d^2(tk)}{dz^2} - \frac{2}{tk}\left(\frac{dR_i}{dz} + \eta\frac{d(tk)}{dz}\right)\frac{d(tk)}{dz} - \frac{1}{\left((\eta)tk + R_i\right)}\right)\frac{\partial T}{\partial \eta}$$

$$+ \frac{1}{tk^2}\left(1 + \left(\frac{dR_i}{dz} + \eta\frac{d(tk)}{dz}\right)^2\right)\frac{\partial^2 T}{\partial \eta^2} - \frac{2}{tk}\left(\frac{dR_i}{dz} + \eta\frac{d(tk)}{dz}\right)\frac{\partial}{\partial \eta}\left(\frac{\partial T}{\partial z}\right). \qquad (26)$$

This function is valid for both normal axisymmetric geometries in which the superfluous terms drop out as well as arbitrary axisymmetric geometries.

4. DERIVATION OF 2-D AXISYMMETRIC ARBITRARY GEOMETRY CONDUCTION EQUATION FOR ANISOTROPIC MATERIALS

Equation 1, rewritten here, is general enough to use as the starting point for this derivation.

$$dV\frac{\partial U}{\partial t} = \frac{\partial}{\partial r}\left((krd\theta dz)\frac{\partial T}{\partial r}\right)dr + krd\theta drdz\frac{\partial^2 T}{\partial r^2}$$

$$+ \frac{\partial}{\partial z}\left((krd\theta dr)\frac{\partial T}{\partial z}\right)dz + kd\theta drdz\frac{\partial^2 T}{\partial z^2}, \qquad (27)$$

The principle difference from the previous derivation is that now the conductivity is functionally dependent, which disallows it to be simply pulled out of the spacial derivatives. Expanding the newly dependent portions of the diffusion terms reveals the difference,

$$\frac{\partial}{\partial z}\left(krd\theta dz\right) = rd\theta dz\frac{\partial k}{\partial z},$$

$$\frac{\partial}{\partial r}\left(krd\theta dz\right) = rd\theta dz\frac{\partial k}{\partial r} + kd\theta dz. \qquad (28)$$

The conductivity could be assumed to be only temperature dependent at this point and the chain rule applied to transform the spacial gradients of the conductivity to temperature gradients. This would enable some simplification but may reduce the generality and treatable anisotropic behavior of certain materials. Generality will be maintained by directly substituting these terms into equation (27) without the aforementioned assumption which results in

$$r d\theta dr dz \frac{\partial U}{\partial T}\frac{\partial T}{\partial t} = \left(r d\theta dz \frac{\partial k}{\partial r} + k d\theta dz \right)\left(\frac{\partial T}{\partial r}\right) dr + k r d\theta dr dz \frac{\partial^2 T}{\partial r^2}$$

$$+ r d\theta dr \frac{\partial k}{\partial z}\frac{\partial T}{\partial z} dz + k d\theta dr dz \frac{\partial^2 T}{\partial z^2} . \qquad (29)$$

Dividing through by the volume and the conductivity leaves

$$\frac{\rho C_p}{k}\frac{\partial T}{\partial t} = \frac{1}{k}\frac{\partial k}{\partial r}\frac{\partial T}{\partial r} + \frac{1}{k}\frac{\partial k}{\partial z}\frac{\partial T}{\partial z} + \frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} . \qquad (30)$$

This is essentially the same function as before except for the obvious addition of the first two terms on the right-hand side. Transforming these terms into generalized coordinates requires the use of the derivatives defined in the previous section

$$\frac{\partial k}{\partial r}\frac{\partial T}{\partial r} = \left(\frac{1}{tk}\right)^2 \frac{\partial k}{\partial \eta}\frac{\partial T}{\partial \eta} ,$$

and

$$= \left(\frac{L}{tk}\right)^2 \frac{\partial k}{\partial \eta}\frac{\partial T}{\partial \eta} + \frac{\partial k}{\partial \xi}\frac{\partial T}{\partial \xi} - \frac{L}{tk}\left(\frac{\partial k}{\partial \xi}\frac{\partial T}{\partial \eta} + \frac{\partial k}{\partial \eta}\frac{\partial T}{\partial \xi}\right) . \qquad (31)$$

Inserting these terms transformed using equation (8) into the right-hand side of equation (26) produces the following generalized 2-D axisymmetric conduction equation with variable physical properties

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2} + \frac{L}{tk^2}\frac{d(tk)}{dz}\left(\frac{\partial T}{\partial \eta}\right)$$

$$- \frac{1}{tk}\left(\frac{d^2 R_i}{dz^2} + \eta\,\frac{d^2(tk)}{dz^2} - \frac{L}{tk}\frac{d(tk)}{dz}\right)\frac{\partial T}{\partial \eta}$$

$$+ \left(\frac{L}{tk}\right)^2 \frac{\partial^2 T}{\partial \eta^2} - \frac{L}{tk}\left[\frac{\partial}{\partial \eta}\left(\frac{\partial T}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial \eta}\right)\right]$$

$$+ \left(\frac{1}{tk}\right)^2 \frac{\partial^2 T}{\partial \eta^2} + \frac{1}{tk\big((\eta)tk + R_i\big)}\frac{\partial T}{\partial \eta}$$

$$+ \left(\frac{L}{tk}\right)^2 \frac{\partial k}{\partial \eta}\frac{\partial T}{\partial \eta} + \frac{\partial k}{\partial z}\frac{\partial T}{\partial z} - \frac{L}{tk}\left(\frac{\partial k}{\partial z}\frac{\partial T}{\partial \eta} + \frac{\partial k}{\partial \eta}\frac{\partial T}{\partial z}\right). \qquad (32)$$

Substituting in L and regrouping produces

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2}$$

$$- \frac{1}{tk}\left(\frac{d^2 R_i}{dz^2} + \eta\,\frac{d^2(tk)}{dz^2} - \frac{2}{tk}\left(\frac{dR_i}{dz} + \eta\,\frac{d(tk)}{dz}\right)\frac{d(tk)}{dz} - \frac{1}{\big((\eta)tk + R_i\big)}\right)\frac{\partial T}{\partial \eta}$$

$$+ \frac{1}{tk^2}\left(1 + \left(\frac{dR_i}{dz} + \eta\,\frac{d(tk)}{dz}\right)^2\right)\frac{\partial^2 T}{\partial \eta^2} - \frac{2}{tk}\left(\frac{dR_i}{dz} + \eta\,\frac{d(tk)}{dz}\right)\frac{\partial}{\partial \eta}\left(\frac{\partial T}{\partial z}\right) + \frac{\partial k}{\partial z}\frac{\partial T}{\partial z}$$

$$+ \frac{1}{tk^2}\left(\frac{dR_i}{dz} + \eta\,\frac{d(tk)}{dz}\right)^2 \frac{\partial k}{\partial \eta}\frac{\partial T}{\partial \eta} - \frac{1}{tk}\left(\frac{dR_i}{dz} + \eta\,\frac{d(tk)}{dz}\right)\left(\frac{\partial k}{\partial z}\frac{\partial T}{\partial \eta} + \frac{\partial k}{\partial \eta}\frac{\partial T}{\partial z}\right). \qquad (33)$$

It is understood that the conductivity k and the specific heat are usually functions of temperature. However, given new material processing practices, gradient or grossly anisotropic properties may result. This form of the equation is general enough to handle either thermal or spacial deviations of the conductivity and specific heat.

5. SUMMARY

A generalized form of the 2-D arbitrary geometry axisymmetric heat conduction equation has been derived from first principles. This has been further generalized by allowing anisotropic and temperature dependent behavior of the material properties. The relation provided could be incorporated into current and future heat conduction models in ARL. Extension of this derivation to three dimensions requires the addition of the azimuthal spacial term and expanding, with proper substitutions similar to the radial terms.

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